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Study of Energy States and U(5) – O(6) Transitional Symmetry of Even–Even104 –108Cd Isotopes by Interacting Boson Model(IBM-1).

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Abstract

In this study we calculated the energy levels of low lying structure for 104 – 108 Cd isotopes and the reduced transition B(E2) of even – even Cd nuclei for A=104,106, 108 by using" the interaction boson model IBM-1" and compared with experimental values .The ratio R(4/2) for the energy levels for 4_1^+ and 2_1^+ states were also calculated for those isotopes .The 104 – 108 Cd nuclei in " U(5) – O(6) transitional symmetry" were studied .The contour plots of the potential energy surfaces (P E S) was calculate for the isotopes above .

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1. Introduction

Digital A ((nuclear)) model was proposed by Arima and Iachello in 1974 [1]. This model was able to describe the characteristics of energy levels and nuclear structure of even – even nucleus [1,2,3] , with medium and heavy mass numbers by pairs of nucleons outside the closed shell that has the magical numbers (2,8,20,28,50,82,126,184) where it has been treated as proton and neutron boson degrees of freedom are not distinguished . This model is called the first interactive boson model IBM-1. The order of these particles (bosons) is important for low levels of even – even nucleus , where the particles are treated as pairs of total angular momentum equal to j = 0 or j = 2 and these pairs are treated as bosons and the proton boson and neutron boson which has angular momentum j= 0 is symboled (s_π , s_ν) respectively [4,5,6] .

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The nutrons of 104 – 108 Cd (Z = 48) nucleus are near to magic number 50 [7,8] . In this work , we study the transitional symmetry U(5) – O(6) and B(E2) values for even – even 104 – 108 Cd isotopes using IBM-1 [1] .

2. Theory and Method of Calculation

2.1. Yarest -State

The energy states are calculated using the following eq. : Re [9, 10].

$$H = \sum_{j=1}^{\aleph} \varepsilon j + \sum_{i<j}^{\aleph} \nu ij$$

Whereas ε is the boson energy and νij is the interaction between bosons i and j. The Hamiltonian is given by [9]

H= εnd + a0 pp + a1LL + a2 QQ + a3 T3T3+ a4T4T4
Where a0, a1, a2, a3 and a4 are parameters used in IBM-1 to determine the Hamiltonian function [11,12,13].
nd : d-bosons operator .
p : operator of pairing among bosons .
L :Angular momentum operator.

Q :Quadrupole operator .
T3 :Octopule oparator .
T4 :Hexadecapole oparator .
The Hamiltonian of the symmetry U(5) _O(6) is given by [12, 14]

H= εnd + a0 pp + a1LL

2 . 2 Reduce Transition Probabilities B(E2)

The reduced matrix elements of the E2 operator T^{E2} "have the form [15 , 16 , 17] .

$$T^{E2} = \alpha_2 \, [d^{+} s + s^{+} d]^{(2)} + \beta_2 \, [d^{+} d]^{(2)}$$

Where (s^+ , d^+) and (s , d) are creation and annihilation operators for s and dbosons , respectively , while α2 and β2 are parameters [15,17]

$$B\ (E2\ , J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \ |\langle J_f \,||\, T^{E2} \,||\, J_i \rangle|^2$$

3 . Results and Discussion

3 . 1 The RL/2 classification

In order to present a comprehensive description for the application of interacting boson model IBM-1 , calculations are firstly introduced by using the IBM-1 for 104 – 108 Cd isotopes that are related to dynamical symmetry U(5)–O(6) see Table 1. Where table 1 shows the rates of the experimental and theoretical energy "R8/2 =E(8_1^+)/E(2_1^+), R6/2 =E(6_1^+)/E(2_1^+) ,R4/2 =E(4_1^+)/E (2_1^+)" for even – even isotopes (102 Cd ,106 Cd ,108 Cd)this rates determine the symmetry for each isotopes .

The rates of the E(4_1^+) / E (2_1^+), E (6_1^+) /E (2_1^+) and (8_1^+)/E (2_1^+), values as a function of neutron numbers of Cd neuclei for experimental and theory values are present in figure 1 .

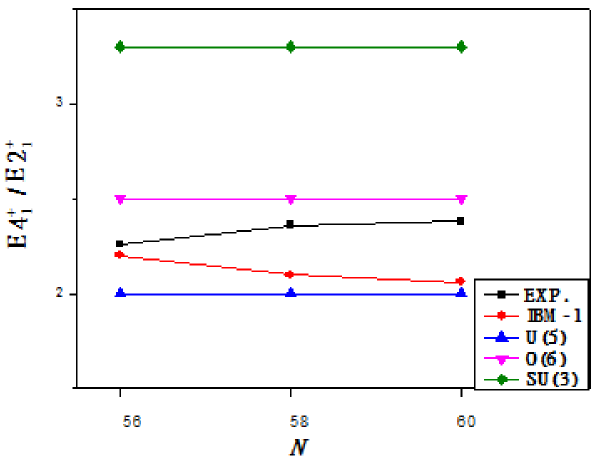

Figure 1a.

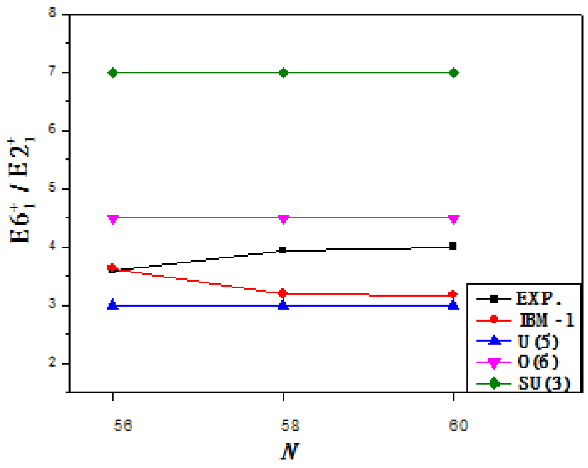

Figure 1b.

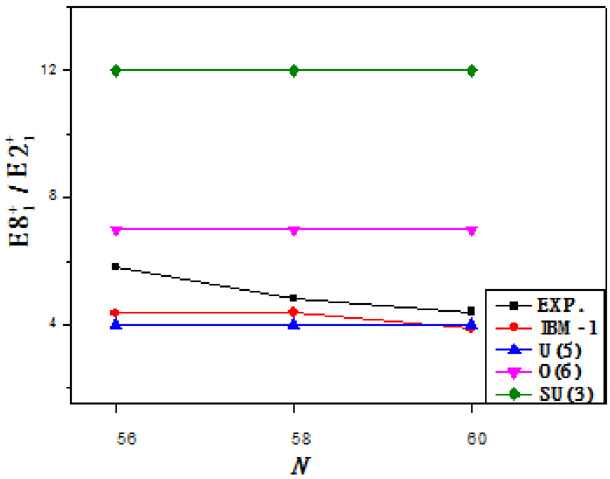

Figure 1c.

Figure 1. E (L_1^+)/ E (2_1^+) values as a function of neutron numbers of 104 – 108 Cd neuclei for experimental values .

From figure 1 , we can see that $^{104-108}$Cd isotopes are related the U(5) – O(6) transitional symmetry . Table 2 shows the calculated values of the Hamiltonian equation parameters using equation. It can be seen that the Hamiltonian equation operator depends on the number of BOS in this model .

Energy levels of even – even isotopes (104 Cd , 106 Cd and 108 Cd) have been classified according to the three bands (g , β and ɤ– , bands) . Table 3 shows values of band's energies for each isotopes with the comparison of them with some available experimental values . It can be shown the accordance in the sequence of energy levels for each band with the ideal sequence for ground – band 0^+, 2^+ ,4^+,6^+,8^+ and beta – band 0^+, 2^+ ,4^+,6^+,8^+ and for gamma – band 2^+, 3^+ ,4^+,5^+, 6^+, 7^+ ,8^+. We can notice an agreement between practical and theoretical values through figures 2 , 3 , 4 . These figures show energy levels for each isotope in a way that fulfills the identical form for energy bands.

Table 1.The rates of experimental and theoretical energy "E (8_1^+) / E (2_1^+) , E (6_1^+) /E(2_1^+) , E(4_1^+) / E (2_1^+) " for even – even isotopes (104 Cd , 106 Cd , 108 Cd) .

Isotopes	EXP.	IBM-1	EXP.	IBM-1	EXP.	IBM-1
^{104}Cd	2.26	2.2	3.6	3.64	4.4	3.87
^{106}Cd	2.36	2.1	3.94	3.2	4.82	4.38
^{108}Cd	2.38	2.06	4.01	3.18	5.82	4.36

Table 2. The calculated values of initial parameters for Hamiltonian function operator for even – even isotopes (104 Cd , 106 Cd, 108 Cd) by using the program (IBM-1) .

Isotopes	N	EPS	p.p	I.I	Q.Q	T3.T3	T4.T4
104 Cd	4	0.5007	- 0.0023	0.016	0.0	0.0	0.0
106 Cd	5	0.6007	- 0.0123	0.006	0.0	0.0	0.0
108 Cd	6	0.631	- 0.0123	0.006	0.0	0.0	0.0

Table 3.The values of the energy bands (g, β, ɤ)(pw) are compared with the experimental values for the isotopes 104 Cd , 106 Cd and 108 Cd .Note : read I^+ for grounds (g– band) or beta band (β – band) ,and read (I^+) for gama bands (ɤ–band).

Isotopes	Dynamical Symmetry	Band	g – band or β - band (ɤ – band) (Mev) [18]							
			$0^+(0^+)$	$2^+(2^+)$	$4^+(3^+)$	$6^+(4^+)$	$8^+(5^+)$	(6^+)	(7^+)	(8^+)
$^{104}_{56\ 48}$Cd	U(5)-O(6)	g -(pw)	0.0	0.6	1.327	2.181	2.319			
		g-Exp.	0.0	0.658	1.492	2.370	2.903			
		β-(pw)	1.001	1.103	1.829	2.682	2.682			
		β- Exp.				2.435				
		ɤ -(pw)	1.509	1.597	1.701	2.319	2.49	2.32		3.16
		ɤ -Exp.								3.30
106 Cd 58 48	U(5)-O(6)	g -(pw)	0.0	0.662	1.365	2.111	2.898			
		g-Exp.	0.0	0.632	1.493	2.491	3.044			
		β- (pw).	1.215	1.853	2.531	3.251	3.251			
		β- Exp.	1.795	2.378	2.304	2.924				
		ɤ -(pw)	1.859	1.281	1.931	1.979	2.646	2.72	3.4 0	3.50
		ɤ -Exp.	2.144			2.104	2.33	2.50		3.37
$^{108}_{60\ 48}$Cd	U(5)-O(6)	g -(pw)	0.0	0.692	1.426	2.201	3.019			
		g-Exp.	0.0	0.633	1.508	2.541				
		β-(pw).	1.276	1.943	2.652	3.403	3.403			
		β- Exp.		2.162	2.645					
		ɤ -(pw)	1.949	1.342	2.021	2.069	2.767	2.84	3.5 5	3.65
		ɤ -Exp.	1.913	1.601	2.145	2.239	2.565	2.81		

Table 4. The values of parameters for reduced matrix elements. $\langle I_f \| T^{E2} \| I_i \rangle$ for isotopes 104Cd ,106Cd and 108Cd by using (IBST) program .

Isotopes	N_π	N_ν	N	α_2	β_2
^{104}Cd	1	3	4	0.18	– 0.265
^{106}Cd	1	4	5	0.17	– 0.265
^{108}Cd	1	5	6	0.15	– 0.265

3.3 Reduced Transition Probabilities B (E2)

The calculated values of α2 and β2 parameters in IBM-1 model (Eq. 4) are summarized in table 4.
The experimental values of " B(E2)" are adoped in the calculations .
Table 4 shows the relation between the parameters (α_2 , β_2) and the number of bosons. It is illustrated that the values of α_2 parameters decrease when the number of boson increase . The calculated values of B(E2) are compared with the experimental values for selected even – even isotopes . The present results are close to the experimental values. This comparison is shown in table 5.

Table 5. The present work of B (E2) and compared with experimental values for isotopes 104 Cd , 106 Cd and 108 Cd .

Isotopes 104 Cd	Dynamical Symmetry "U(5) – O(6)"	
$I_i^+ - I_f^+$	B(E2) (eb)^2	
	IBM-1 (pw)	EXP.[18]
$2_1^+ \rightarrow 0_1^+$	0.355	0.300
$2_2 \rightarrow 2_1^+$	0.572	
$4_1^+ \rightarrow 2_1^+$	0.572	0.578
$6_1^+ \rightarrow 4_1^+$	0.600	0.715
$6_2^+ \rightarrow 4_1^+$	0.0001	
$2_3^+ \rightarrow 0_1^+$	0.000	
$2_3^+ \rightarrow 0_2^+$	0.306	
$4_3^+ \rightarrow 2_1^+$	0.000	
$2_4^+ \rightarrow 0_1^+$	0.000	
$2_1^+ \rightarrow 0_1^+$	0.316	0.251
$2_2^+ \rightarrow 2_1^+$	0.510	0.589
$4_1^+ \rightarrow 2_1^+$	0.510	0.141
$6_1^+ \rightarrow 4_1^+$	0.577	
$6_2^+ \rightarrow 4_1^+$	0. 001	
$2_3^+ \rightarrow 0_1^+$	0.000	
$2_3^+ \rightarrow 0_2^+$	0.273	
$4_3 \rightarrow 2_1^+$	0.000	
$2_4^+ \rightarrow 0_1^+$	0.000	
$2_1^+ \rightarrow 0_1^+$	0.247	0.248
$2_2^+ \rightarrow 2_1^+$	0.397	0.388
$4_1^+ \rightarrow 2_1^+$	0.397	0.161
$6_1^+ \rightarrow 4_1^+$	0.449	
$6_2^+ \rightarrow 4_1^+$	0. 001	
$2_3^+ \rightarrow 0_1^+$	0.000	
$2_3^+ \rightarrow 0_2^+$	0.212	
$4_3^+ \rightarrow 2_1^+$	0.000	
$2_4^+ \rightarrow 0_1^+$	0.000	

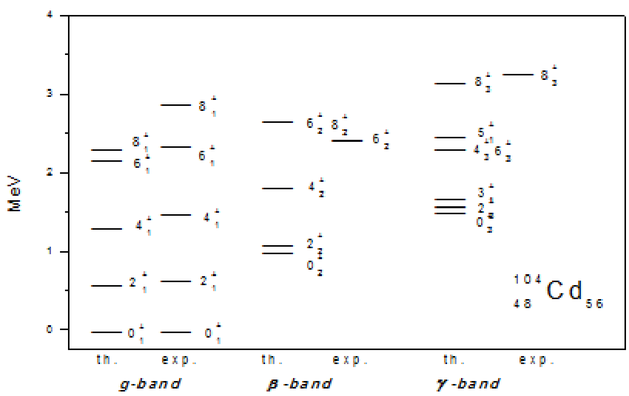

Figure 2. The energy levels for ^{104}Cd [18].

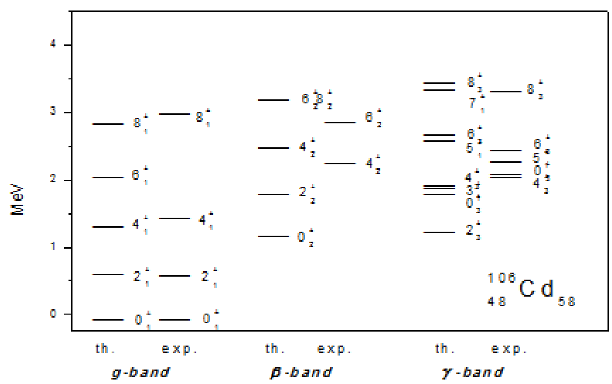

Figure 3. The energy levels for 106 Cd [18] .

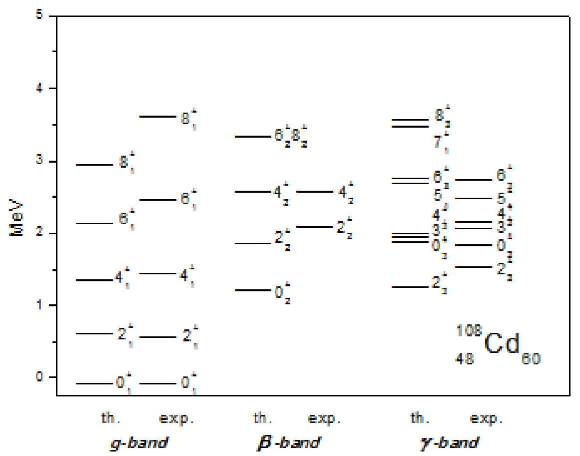

Figure 4 .The energy levels for ^{108}Cd [18] .

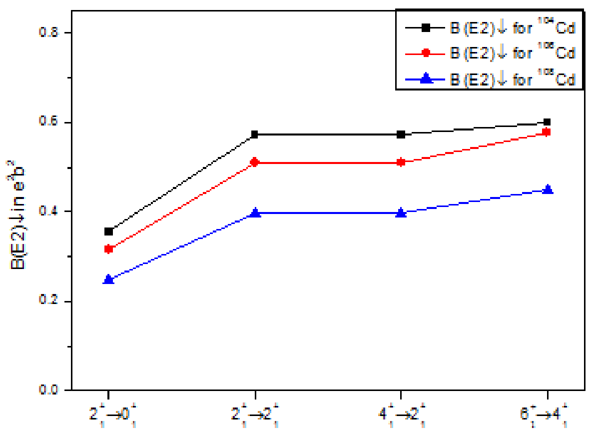

Figure 5. The electric quadrupole transitions as a function of transitions $(2_1^+ \rightarrow 0_1^+)$,$(2_2^+ \rightarrow 2_1^+)$, $(4_1^+ \rightarrow 2_1^+)$ and $(6_1^+ \rightarrow 4_1^+)$ for 104 – 108Cd nuclei

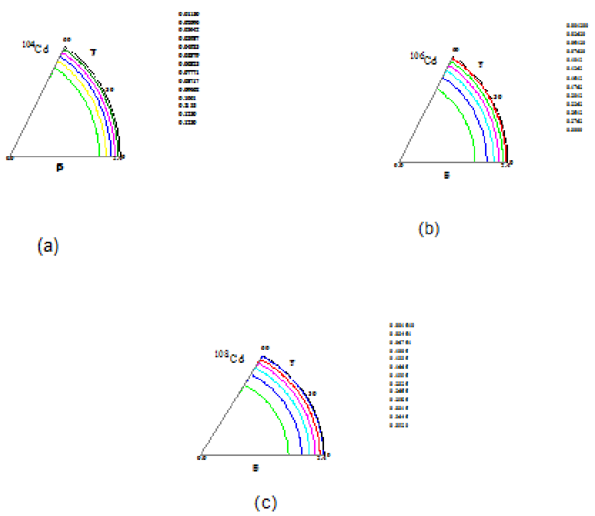

Figure (6) . potential energy surface for even 104 Cd 106 Cd and 108Cd isotopes .

3.4 Potential Energy Surface (P E S)

By the skyrme mean field method was mapped onto the PES of the " IBM Hamiltonian" [19,20]. The expectation value of the "IBM-1 Hamiltonian" with the coherent state $|N,\beta,\gamma\rangle$ is used to create the" IBM energy Surface" [21,22]. The state is a product of the boson creation operator(νc .) with .

$$| N,\beta,\gamma \rangle = \frac{1}{\sqrt{Ni}} (b_c^{+})^{N} | 0 \rangle$$

$$b_c^{+} = (1+\beta^2)^{-1/2} \{S^{+} + \beta [\cos\gamma (d_0^{+}) + \sqrt{1/2} \sin\gamma(d_2^{+} + d_{-2}^{+})]\}$$

The energy surface as a function of β and γ , has been given [22] .

$$E (N,\beta,\alpha) = \frac{N\varepsilon d\beta^2}{(1+\beta^2) + \frac{N(N-1)}{(1+\beta^2)^2}(\alpha_1\beta^4 + \alpha_2\beta^3 \, \mathrm{Cos}3\gamma + a3\,\beta^2 + \alpha_4)}$$

The calculated potential energy surface, E (N , β , γ) of 104 Cd 106 Cd and 108Cd are shown in figure 6.

4. Conclusion

The" interacting boson model" was used to study the energy states of 104 – 108 Cd isotopes for positive parity , the "B(E2) transitions " and the "potential energy surface" for the isotopes above was calculated .It is seen that the energy states and the probability B(E2)of those isotopes are given a good agreement with the experimental values. The even Cadmium 104 – 108 Cd nuclei in "U(5) – O(6) symmetry" were also studied.

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